A COMPATIBLE ESTIMATION MODEL OF STEM VOLUME AND TAPER FOR *Acacia mangium* Willd. PLANTATIONS

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Received: 10 July 2015, Revised: 28 March 2016, Accepted: 8 April 2016

A COMPATIBLE ESTIMATION MODEL OF STEM VOLUME AND TAPER FOR *Acacia mangium* Willd. PLANTATIONS. This study describes the establishment of a compatible volume estimation model for *Acacia mangium* Willd on the basis of 279 felled sample trees collected from the *A. mangium* plantation stands in South Sumatra, Indonesia. The model comprises of a total volume model and a stem taper model, which is compatible in the sense of the total volume obtained by integration of the taper model being equal to that computed by the total volume model. Several well-known total volume functions were evaluated including constant form factor, combined variable, generalized combine variable, logarithmic, generalized logarithmic and Honer transformed variables. A logarithmic model was determined to be the best and was then used as the basis for deriving the taper model. Appropriate statistical procedures were used in model fitting to account for the problems of heteroscedasticity and autocorrelation that are associated with the construction of volume and taper functions. The simultaneous fitting method of the Seemingly Unrelated Regression (SUR) improved the parameter estimates and goodness-of-fit statistics while ensuring numeric consistency among the component models and reducing the total squared error obtained by an independent fitting method. The developed model can be used to estimate total stem volume, merchantable volume to any merchantability diameter limit at any height, and (possibly) height of any diameter based on only easily measurable parameters such as diameter at breast height and total tree height for the species analysed.

Keywords: *Acacia mangium*, compatible volume, estimation model, taper, timber volume

MODEL YANG KOMPATIBEL UNTUK PENDUGAAN VOLUME DAN TAPER BATANG HUTAN TANAMAN Acacia mangium Willd. *Tulisan ini mempelajari penyusunan model penduga volume yang kompatibel untuk jenis* Acacia mangium *Willd. berdasarkan data dari 279 pohon contoh yang ditebang dari areal tegakan hutan tanaman* A. mangium *di Sumatera Selatan, Indonesia. Model ini terdiri dari model volume total dan model taper batang, yang kompatibel dalam arti volume total yang diperoleh dari integrasi model taper sama dengan volume yang dihitung dengan model volume total. Beberapa fungsi persamaan umum volume total diuji, termasuk faktor bentuk konstan, variabel gabungan, variabel gabungan umum, logaritmik, logaritmik umum dan variabel Honer yang ditransformasi. Hasil pengujian menunjukkan bahwa model logaritmik merupakan model terbaik dan dipilih sebagai dasar untuk menurunkan model taper. Prosedur statistik yang sesuai digunakan dalam penyusunan model untuk mengatasi masalah heteroskedastisitas dan autokorelasi yang berkaitan dengan fungsi persamaan volume dan taper. Metode* fitting *secara simultan dari* Seemingly Unrelated Regression *(SUR) menghasilkan estimasi parameter dan statistik kelayakan model yang lebih baik dibandingkan dengan metode* fitting *secara independen dengan tetap menjamin konsistensi numerik diantara model-model komponen dan mengurangi total kuadrat error. Model yang dikembangkan dapat digunakan untuk menduga volume batang total, volume kayu komersial sampai ke batas diameter tertentu yang dapat diperdagangkan, diameter pada setiap ketinggian, dan (memungkinkan) tinggi dari setiap diameter, hanya berdasarkan parameter yang mudah terukur seperti diameter setinggi dada dan tinggi pohon total untuk jenis yang dianalisis.*

Kata kunci: Acacia mangium, *volume yang kompatibel, model pendugaan, taper, volume kayu*

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 doi: http://dx.doi.org/10.20886/ijfr.2016.3.1.49-64

I. INTRODUCTION

Acacia mangium Willd. has been the main species planted in the industrial forest plantations in Indonesia. The wood of *A. mangium* has properties that potentially make it acceptable for a wide range of end-uses, including pulp, veneer and plywood, as well as for sawn timber and other products (Abdul-Kader & Sahri, 1993; Krisnawati, Kallio, & Kanninen, 2011). The wood harvested from *A. mangium* plantations may be merchandised to satisfy the demand for a variety of products. Consequently, the predictions of these product volumes are often more important than total tree volumes. As the standard of merchantability may also change for the pulp and paper industries and the sawmills that may utilise the wood, it is desirable to have a volume model that can predict volume to any specified upper stem diameter, rather than total stem volume model which does not give quantitative information on the amount of wood specified for any particular utilisation standard.

In the development of stem volume estimation models, there are three general methods that can be used to estimate stem volume to any merchantable limit. The first approach is to develop a model for predicting stem volume to a fixed top diameter (Bi & Hamilton, 1998; Tewari & Kumar, 2003; Krisnawati & Bustomi, 2004). This is effective but inflexible if merchantability standards change. The second approach is to develop a volume ratio model that predicts merchantable volume to any specified height limit as a percentage of total stem volume (Reed & Green, 1984; Bi, 1999; Teshome, 2005). The third approach is to develop a stem taper model and obtain estimates of the merchantable volume through integration (Jiang, Brooks, & Wang, 2005; Özçelik & Brooks, 2012; Navar, Rodriguez-Flores, F. J. & Dominguez-Calleros, 2013). Both the second and third approaches eliminate the need for separate volume models for differing merchantability standards as in the first approach. The second approach is easy to use and develop; however, the third approach is generally preferred as this also allows estimation of diameter at a given height (Dieguez-Aranda, Castedo-Dorado, Alvarez-Gonzalez, & Rojo, 2006). The third approach has been considered to be most accurate for estimating the volume to any merchantable limit (Kozak, 2004; Jiang et al., 2005).

A wide range of taper models exists in the literature (e.g. Fang & Bailey, 1999; Fang, Borders, & Bailey, 2000; Sharma & Zhang, 2004; Jiang et al., 2005; Özçelik & Brooks, 2012; Navar, Rodriguez-Flores, F. J. & Dominguez-Calleros, 2013), but their use in Indonesia is still not quite common. Different standard volume models and volume tables are still the most common tools used for estimating the volume of the tree species in Indonesia. The prediction of merchantable volume is usually accomplished by fitting a separate regression model for each merchantability limit (i.e. the first approach). Thus, for a single tree population of a species, there may be three different models for predicting three different merchantable volumes, say, to 4 cm, 7 cm and 10 cm upper stem diameters. The development of separate models for each set of merchantability limits may not only require considerable effort but also the models may not perform satisfactorily when considered together. A few studies have been conducted in Indonesia on the development of taper models for other species (Krisnawati & Wahjono, 2003; Harbagung & Krisnawati, 2009); however, additional work is needed in this area to improve volume estimation.

For *A. mangium* plantations, some volume tables and models with fixed merchantability limits (i.e. 4 cm and 7cm) have been developed in different regions in Indonesia, as summarised in Krisnawati et al. (2011). These include those developed by Sumarna and Bustomi (1986), Bustomi (1988), Wahjono, Krisnawati, and Bustomi (1995), and Krisnawati, Wahjono, and Iriantono (1997) but they may not be sufficient for estimating volume in the currently changing product and market conditions. In addition, they were developed using data with very limited range of tree sizes and ages (e.g. 5 years only). A more flexible and better model therefore needs to be developed for *A. mangium* plantations that covers a wider range of ages and tree sizes and allows prediction of merchantable volume at any specified upper stem diameter.

Ideally, a tree volume estimation model should be compatible, in which the tree volume calculated by a total volume model should be equal to that computed by integration of the stem taper model from the ground to the top of the tree (Clutter, 1980). A simple method that results in compatible, accurate predictions of stem volume and taper of the tree would thus be very useful for practical forest management. The objective of this study was to develop a compatible estimation model of stem volume and taper for *A. mangium* plantations which provide the best possible fit for total stem volume and taper models. The procedure of how to derive a compatible taper and merchantable volume model is described hereafter.

II. MATERIAL AND METHOD

A. Study Site

This study was conducted in the industrial forest plantation area of PT. Musi Hutan Persada in South Sumatra, Indonesia, with *A. mangium* as the main species planted. The overall study site is topographically located at an attitude ranging from approximately 60 to 200 m above sea level. The topography is mostly flat to moderately undulating (0 - 8% in slope) but in some areas is rolling (8 - 15% in slope).

In general, the climate condition in South Sumatra is well suited to *A. mangium* plantations, with even rainfall distribution and relatively constant, warm temperature. In particular, the study area has a lowland humid environment with the average daily temperature of 29°C, ranging from 22 to 33°C. The average relative humidity varies from 56% in the dry season to 81% in the rainy season, with the average annual rainfall ranges from 1890 to 3330 mm.

The soils are derived mainly from sedimentary rocks consisting of tuff, sandy tuff, sandstone and claystone, with a very small portion from volcanic materials. The majority of the soils belong to red-yellow podsolic group (ultisol and oxisol), and are inherently acid and poor in nutrients and have low pH value and low base saturation (Hardiyanto, Anshori, & Sulistyono, 2004).

B. Data Description

Data for this study were obtained from 279 sample trees harvested from *A. mangium* unthinned stands with the planting spacing of 3x3 m. The trees were selected to represent the range of ages, diameter at breast height (DBH) and total tree height. The ages of the sample trees averaged 5.6 years and ranged from 2 to 9 years; DBH averaged 16.3 cm and ranged from 5.7 to 28.8 cm; and total tree height averaged 19.3 m and ranged from 4.7 to 31.5 m. Size distribution of the sample trees based on both

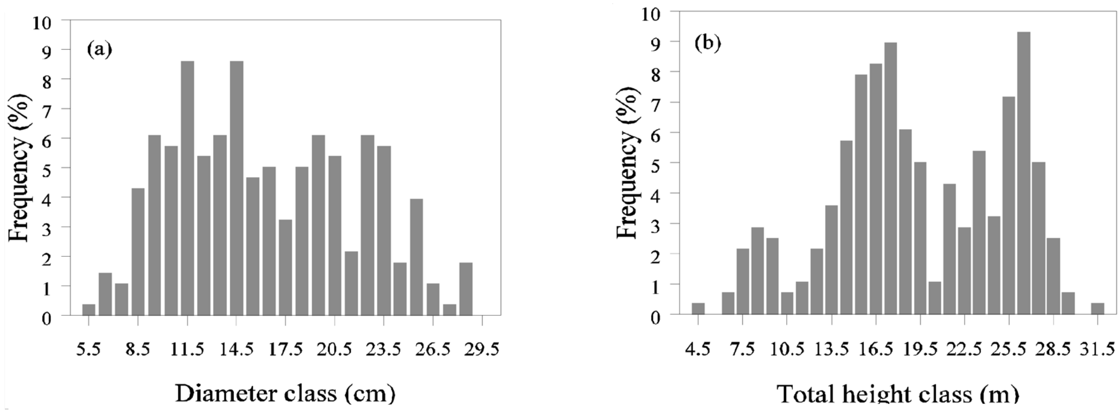

Figure 1. Frequency of the 279 sample trees (in percent) based on DBH (diameter) class (a) and total tree height class (b)

DBH and total tree height classes are shown in Figure 1. The 279 sample trees were split randomly into two data sets by age and 1-cm DBH classes at a ratio 3:1 (209 sample trees were used for model fitting and 70 sample trees were used for model validation).

Before felling, each sample tree was measured for DBH, and after felling, for total tree height. Stump heights were measured and averaged about 10 cm (0.1 m). The felled trees were then cut into 1 m sections, starting from the stump to a top diameter of approximately 4 cm. Each section was measured for diameter over and under bark at the large- and small-end of the sections. The bark thickness was measured to differentiate between the volume to be harvested (over bark) and the volume available for utilization (under bark). From the 279 sample trees, there were 4538 sections available with the number of sections varying from 4 to 26 per tree.

Volumes for each section of each sample tree were calculated using standard formulae applicable to typical tree shapes. The sectional volumes from stump cut to tip were determined using Smalian's formula assuming a frustum of a second degree paraboloid while the volume of the top section was calculated assuming the tip as a cone (Husch, Beers, & Kershaw, 2003). Summation of the volumes of each section gave total volume of each sample tree. Summation from the base to any section provided the merchantable volume to the specific small-end diameter (top diameter) of that section. These volumes are referred to as the "true" stem volume of the tree, although they may differ from the real volume as would be obtained through water displacement methods (e.g. Martin, 1984).

C. Compatible Stem Volume and Taper Models

Compatibility in this study means that the total stem volume, obtained from the volume model is equal to the stem volume derived by integrating the taper model for all trees with the same diameter at breast height and total tree height. There are two approaches for constructing a compatible model: the volume-based model and the taper-based model (Munro & Demaerschalk, 1974). The volume-based model fits a volume model, and then derives the corresponding stem taper model based on the volume model. The coefficients of the taper model are conditioned such that volume, obtained by the volume model is equal to the volume derived by integrating the taper model for all trees. Examples of such an approach are the models developed by Byrne and Reed (1986), and Mc Tague and Bailey (1986). The taper-based model develops a taper model, and the compatibility of the model is ensured by imposing the condition on the coefficients so that integration of the taper model provides the total volume of the tree. Examples of such an approach are the models of Fang and Bailey (1999), and Fang et al. (2000).

In this study, the volume-based model was applied. This approach was selected due to lower biases (Munro & Demaerschalk, 1974) and was more tractable via a geometric approach (Byrne & Reed, 1986). The taper model was derived from the total stem volume model, applied to the tree from the top to down and constrained to predict the same total stem volume when integrated as a direct volume prediction method for the total stem. This constraint was imposed by defining the limits of integration of the taper and total volume models. Stem diameter is equal to zero when distance from the top of the tree to the merchantability limit is equal to total tree height, ensuring compatibility between the merchantable and total volume models.

D. Development of Total Stem Volume Model

To determine the most appropriate model for estimating the total stem volumes both for over bark (*Vob*) and under bark (*Vub*), six individual stem volume model forms that have been used in various studies for predicting individual stem volume in different forest regions and forest types (Clutter, Fortson, Pienaar, Brister, & Bailey, 1983; Husch et al., 2003) were tested.

Table 1. Individual stem volume models tested as the basis for deriving the taper model

Model	Name	Expression
V-1	Constant form factor	$V = a_0 D^2Ht$
V-2	Combined variable	$V = a_0 + a_1D^2Ht$
V-3	Generalized combined variable	$V = a_0 + a_1D^2 + a_2Ht + a_3D^2Ht$
V-4	Logarithmic	$V = a_0D^{a1}Ht^{a2}$
V-5	Generalized logarithmic	$V = a_0 + a_1D^{a2}Ht^{a3}$
V-6	Honer transformed variable	$V = \frac{D^2}{(a_0 + a_1Ht^{-1})}$

In order to choose a model form which provides the most accurate stem volume prediction, all six candidate stem volume models (Table 1) were fitted for over bark and under bark stem volume using least squares regression method. Studentised residual plots showed an uneven spread of residuals for all models, with the variance of residuals increasing with the predicted value, indicating the presence of heteroscedasticity of error. One example of the heteroscedasticity problem is demonstrated for Model V-4 (logarithmic form) in Figure 2(a).

This problem has commonly occurred in other studies fitting regression models to stem volumes (e.g. Williams & Gregoire, 1993; Williams & Schreuder, 1996; Bi & Hamilton, 1998). To correct for the heteroscedastic errors, the solution may be to weight each observation during the fitting process by the inverse of its variance (σ_i^2). Different assumptions about the nature of heteroscedasticity in the construction of volume models may be made; however, the error variance of the *i*th individual is often modelled as a power function of diameter and height, $\sigma_i^2 = (D_i^2 Ht_i)^k$ (Furnival, 1961; Clutter et al., 1983). The most reasonable value of the exponential term *k* should provide the most homogenous studentised residual plot (Huang, 1999), which can be obtained by iteratively testing different values of *k* (e.g. from 0.1 to 2). All models (except Model V-4) were refitted using weighted least squares regressions using a weighting factor, $(D_i^2 Ht_i)^{-k}$, with the optimum values of *k* was selected to the nearest interval of 0.1. In this case, logarithmic transformation to all variables was used to solve the problem

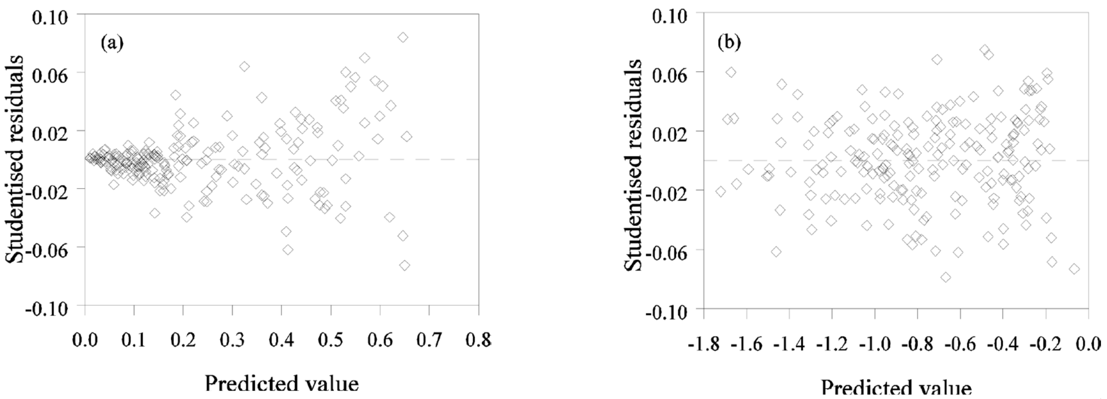

Figure 2. Studentised residuals plotted against predicted values fitted using Model V-4 for over bark volume before transformation (a) and over bark volume after logarithmic transformation (b)

of unequal error variance as well as to attain linearity. Transforming variables in this way produces the model which has a more equal error variance. As illustrated in Figure 2(b), the weighted residuals of Model V-4 show a more even spread when plotted against predicted values.

It should be noted that the use of weighted least squares regression changes the estimates of the parameters and the standard errors of the estimates relative to the values obtained in the absence of weighting (Ratkowsky, 1990). As the usual index of fit could not be applied to compare the candidate total stem volume models that have different dependent variables (after weighting or transformation), Furnival's Index of fit (Furnival, 1961) was used to select the best model for predicting total stem volume. Based on the Furnival's Index of fit, the logarithmic model (Model V-4) was determined to be best for estimating total stem volume both over bark and under bark:

$$\log V = a_0 + a_1 \log D + a_2 \log Ht \qquad (1)$$

A total stem volume model of this type has also been used previously by many studies for estimating stem volumes for *A. mangium* plantations (e.g. Bustomi, 1988; Wahjono et al., 1995; Krisnawati et al., 1997) as well as for other species (e.g. Fang & Bailey, 1999; Fang et al., 2000; Cecilia et al., 2014). This model was also consistently the best in the validation data set. The model was then used as the basis for deriving the taper and merchantable volume models in order to invoke the compatibility constraint.

E. Derivation of Compatible Taper Model

The stem volume model of Eq.(1) can be converted into a stem taper model as follows (e.g. Demaerschalk, 1973; McTague & Bailey, 1986):

$$\log d = b_0 + b_1 \log D + b_2 \log Ht + b_3 \log l \qquad (2)$$

where d is upper stem diameter over bark or under bark (cm), D is DBH (cm), Ht is total tree height (m), l is distance from the tip of the stem to the merchantability limit (m), and b_0, b_1, b_2, b_3 are parameters to be estimated

Mathematically, Eq.(2) can be expressed as:

$$d = 10^{b0} D^{b1} Ht^{b2} l^{b3} \qquad (3)$$

or in a squared form:

$$d^2 = 10^{2b_0} D^{2b_1} Ht^{2b_2} l^{2b_3} \qquad (4)$$

The use of d^2 as dependent variable (Eq.(4)) is mathematically more convenient than d (Eq.(3)) in taper models (Clutter, 1980). The volume of a single tree from the taper model (V_2) of Eq.(4) is calculated by integration of d with respect to l:

$$V_2 = \frac{\pi}{4} 10^{-4} \int_0^{Ht} 10^2 dl \qquad (5)$$

Substitution of Eq.(4) for d^2 into Eq.(5) results in:

$$V_2 = \frac{\pi}{4} 10^{-4} \int_0^{Ht} 10^{2b_0} D^{2b_1} Ht^{2b_2} l^{2b_3} dl \qquad (6)$$

$$V_2 = \left(\frac{\pi}{4} 10^{-4} 10^{2b_0} D^{2b_1} Ht^{2b_2}\right) \int_0^{Ht} l^{2b_3} dl$$

$$V_2 = \left(\frac{\pi}{4} 10^{2b_0 - 4} D^{2b_1} Ht^{2b_2}\right) \frac{1}{2b_3 + 1} \left[l^{2b_3 + 1} \right]_0^{Ht}$$

when $l = Ht$, that is, the distance from the tip to the upper stem diameter is equal to the total tree height, the above model:

$$V_2 = \frac{\pi}{4} 10^{2b_0 - 4} D^{2b_1} Ht^{2b_2} \frac{1}{2b_3} Ht^{2b_3 + 1}$$

$$V_2 = \frac{\pi}{4} 10^{2b_0 - 4} D^{2b_1} Ht^{2b_2 + 2b_3 + 1} \frac{1}{2b_3 + 1}$$

Let, $\frac{\pi}{4} \frac{10^{2b_0 - 4}}{2b_3 + 1} = C$ Eq.(6) then becomes:

$$V^2 = CD^{2b_1} Ht^{2b_2} + 2^{b_3} + 1 \qquad (7)$$

Eq.(7) can be transformed into a logarithmic form:

$\log V2 = \log C + 2b_1 \log D + (2b_2 + 2b_3 + 1)\log Ht$ (8)

Based on the compatibility constraint, that is, V_1 (volume derived from the total volume model, Eq.(1)) is equal to V_2 (volume obtained from the taper model, Eq.(8)), the coefficients of these two models are conditioned such that:

$a_0 = \log C$ (9)

$a_1 = 2b_1$ (10)

$a_2 = 2b_2 + 2b_3 + 1$ (11)

The coefficients of the taper model (Eq.(2)) were expressed in terms of the coefficients of the volume model (Eq.(1)), i.e. a_0, a_1, a_2 and one 'free parameter' (p), which is a unique value for a given set of taper data. This parameter (p) expresses the variability of d along the stem of a tree and depends on the D, Ht and l.

Let $2b_3 + 1 = pa_2$ (12)

Eq.(9) can be rewritten as:

$$a_0 = \log C = \log\left[\frac{\pi 10^{2b_0 - 4}}{4pa_2}\right] \quad (13)$$

Solving for b_0 results in:

$$b_0 = \frac{1}{2}\left[\log\left[\frac{4}{\pi}10^{a_0 + 4} pa_2\right]\right] \quad (14)$$

Eq.(10) can be rewritten as:

$$b_1 = \frac{a_1}{2} \quad (15)$$

Eq.(11) can be rewritten as: (16)

$$b_2 = \frac{a_2(1-p)}{2}$$

The coefficient of b_3 can be derived from Eq.(11):

$$b_3 = \frac{pa2 - 1}{2}$$

F. Estimation of Merchantable Volume Model to Any Specified Upper Stem Diameter

To predict merchantable volume of a tree to any upper stem diameter limit, the compatible taper model (Eq.(3)) was used. Algebraically, Eq.(3) can be rearranged to estimate the distance l from top of the stem to the point of merchantability limit (d): (17)

$$l = 10^{-b_0/b_3} d^{1/b_3} D^{-b_1/b_3} Ht^{-b_2/b_3}$$

The volume to a specific distance l from tip of the stem (V_l) can be calculated as follows: (18)

$$V_1 = CD^{2b_1} Ht^{2b_2} l^{2b_3+1}$$

The compatible taper model after being integrated with respect to l, provides the total volume (Vt) when total tree height Ht equals a given distance l as derived in Eq.(7): (19)

$$V_1 = CD^{2b_1} Ht^{2b_2+2b_3+1}$$

The merchantable volume to any merchantability limit d (V_m) is therefore the difference between V_{ta} and V_l: (20)

$$V_m = V_t - V_l$$

Substituting the right sides of Eqs.(18) and (19) into Eq.(20) results in: (21)

$$V_m = CD^{2b_1} Ht^{2b_2} [Ht^{2b_2+1} - l^{2b_3+1}]$$

Substituting l from Eq.(17) into Eq.(21) provides the merchantable volume to any merchantability limit (d):

$$V_m = CD^{2b_1} Ht^{2b_2}\left[Ht^{2b_3+1} - \left[10^{-b_0\left[\frac{2b_3+1}{b_3}\right]} d^{\left[\frac{2b_3+1}{b_3}\right]} D^{-b_1\left[\frac{2b_3+1}{b_3}\right]} Ht^{-b_2\left[\frac{2b_3+1}{b_3}\right]}\right]\right]$$

G. Parameter Estimation of the Compatible Volume and Taper Models

The compatible model analysed has two components: a total volume model (V = f (DBH, Ht)) and a taper model (d = f (DBH, Ht, l)). The model was composed by the endogenous variables (variables included on the left hand side of equation) V and d, which are assumed to be determined by the model structure and exogenous variables D, Ht and l, which are independent variables. To ensure compatibility between the total volume and the taper models, two methods can be used for fitting the volume-based model in this study. The first method is to fit the total volume model using the total volume observations, and then algebraically solve for the parameters of the taper model based on the previously obtained parameters from the fitted volume model. The second is to estimate all the parameters of the model (both the volume and taper models) simultaneously, in which the parameters were expanded by the compatibility relationship when programming the models prior to fitting.

Estimation of the parameters for both over and under bark measurements was carried out using the MODEL procedure of SAS/ETS model (SAS Institute Inc., 2005). For the first method, ordinary least squares estimation procedure was applied. For the second, the seemingly unrelated regression (SUR) technique was used to fit the model. The SUR technique was considered since both the total volume and taper models seem unrelated (none of the endogenous variables in one equation of the model appears as an independent variable in another equation), but the equations are related through the correlation in the errors. A set of equations that has contemporaneous cross-equation error correlation (i.e. the error terms in the regression equations are correlated) is called a seemingly unrelated regression (SUR) (Judge, Hill, Griffiths, Lütkepohl, & Lee, 1988).

It should be noted that in the compatible model composed by a total volume and a taper model, the number of observations in each model is not equal. There is more than one diameter observation for each tree but only one observation for the total stem volume. However, simultaneous fitting of both models requires the number of observations of the two endogenous variables to be equal. To overcome this problem, a special structure of the data was created by assigning the total volume to each diameter observation on the same tree.

Since the taper data comprise multiple stem diameter measurements along each sample tree, autocorrelations may exist among the residuals, which violates the assumption of independent error terms. To check for the possible autocorrelation, graphs representing residuals versus residuals of the adjacent section within the same tree were examined visually. Appropriate fits for the models with correlated errors were done by including the autoregressive error structure in the MODEL procedure of SAS/ETS system (SAS Institute Inc., 2005).

H. Model Evaluation

Performance of the models of each fitting method was evaluated based on the statistical properties such as asymptotic t-statistics for significance of the parameters, standard error of coefficient (SE), root mean squared error (RMSE), adjusted coefficient of determination (R^2adj) of the model. In addition, the common statistics of average bias (i.e. observed - predicted) and standard error of predictions were calculated to evaluate the performances of the compatible volume and taper models. Since the merchantable volume model was derived from the compatible stem volume and taper model, the possible bias and standard error of the model predictions were also examined. The accuracy of these predictions was evaluated over the entire range of the validation data set. The performance of the models in different parts of the stem and for various sizes of tree (i.e. by DBH and tree height classes) was also examined.

A commonly used method for evaluating prediction accuracy within specified diameter or total height classes is sorting the prediction errors according to DBH or total tree height, then dividing them into intervals of equal width and calculating relevant statistics on bias and standard error of the predictions. These statistics are very important for showing areas or tree size classes for which the compatible volume and taper model and its implied merchantable volume model provide especially good or poor predictions (Kozak & Smith, 1993; Kozak, 2004).

III. RESULT AND DISCUSSION

A. Estimation of the Compatible Volume and Taper Model

Parameters in the compatible model were estimated by two different methods. In the first method the logarithmic total volume model was fitted independently using the total volume observations. Following this, the estimated parameters were substituted into the taper model and the remaining parameter

Table 2. Parameter estimates, standard errors and related fit statistics of "independent" (first method) and simultaneous (second method) fittings of the compatible volume and taper models for both over bark and under bark

Model	Parameter	Estimates	*SE*	*t*	*p*-value	*RMSE*	R^2_{adj}
Independent fitting							
log*Vob*	a_0	-4.2117	0.019	-214.36	0.000	0.033	0.994
	a_1	1.734	0.0268	64.59	0.000		
	a_2	1.081	0.0276	39.10	0.000		
log*dob*	b_0	0.1339	0.007	19.13	0.000	0.040	0.954
	b_1	0.867	0.0085	102.49	0.000		
	b_2	-0.644	0.0102	-63.32	0.000		
	b_3	0.685	0.0031	222.99	0.000		
log*Vub*	a_0	-4.3784	0.025	-177.18	0.000	0.033	0.992
	a_1	1.744	0.0302	57.81	0.000		
	a_2	1.163	0.0296	39.33	0.000		
log*dub*	b_0	0.0413	0.005	8.43	0.000	0.039	0.949
	b_1	0.872	0.0094	92.76	0.000		
	b_2	-0.554	0.0108	-51.18	0.000		
	b_3	0.635	0.0035	182.09	0.000		
Simultaneous fitting							
log*Vob*	a_0	-4.197	0.0071	-590.03	0.000	0.011	0.999
	a_1	1.736	0.0071	322.97	0.000		
	a_2	1.0734	0.00259	414.63	0.000		
log*dob*	b_0	0.14	0.003	40.66	0.000	0.031	0.973
	b_1	0.8678	0.00269	322.97	0.000		
	b_2	-0.6393	0.00154	-414.63	0.000		
	b_3	0.676	0.0028	238.35	0.000		
log*Vub*	a_0	-4.27	0.008	-530.52	0.000	0.012	0.999
	a_1	1.778	0.0064	278.89	0.000		
	a_2	1.04	0.003	343.49	0.000		
log*dub*	b_0	0.099	0.0039	25.37	0.000	0.032	0.967
	b_1	0.889	0.0032	278.89	0.000		
	b_2	-0.6213	0.00181	-343.49	0.000		
	b_3	0.643	0.0033	193.20	0.000		

Notes: *Vob* is stem volume over bark (m^3), *Vub* is stem volume under bark (m^3), *dob* is upper stem diameter over bark (cm) and *dub* is upper stem diameter under bark (cm)

of the model was then estimated. In the second method, all parameters in the model were estimated simultaneously using the SUR technique. In this case, parameters in the total volume model were estimated in such a way that they not only minimised the squared error in the total volume, but also minimised the squared error for the taper model. Due to the longitudinal nature of the taper data used for model fitting, a trend in residuals as a function of residuals of the adjacent section within the same tree was apparent in the models analysed (plots not presented here). This has also been found in similar studies for other species (Rojo, Perales, Sánchez-Rodríguez, Álvarez-González, & Gadow, 2005; Corral-Rivas, Diéguez-Aranda, Rivas, & Dorado, 2007). After correcting for autocorrelation using a first order autoregressive error structure, the trends in residuals disappeared. The final parameter estimates and their corresponding standard errors for the models fitted using the two methods are presented in Table 2.

As can be seen in Table 2, the corresponding parameter estimates from the two methods were very similar and all parameters in the models were found to be significant (*p*-value< 0.01). All parameter estimates are logical and ensure compatibility between the volume and taper models. Goodness-of-fit statistics show that the compatible models fit both volume and upper stem diameter data reasonably well. For example, more than 94% (R^2adj) of the variation about the values of d and V for both over and under bark is explained by the model.

With the "independent" fitting (first method), it is quicker to achieve convergence on the parameter estimates, and may provide the best estimate of the total volume. While independent estimation (for total volume) can lead to more accurate and precise prediction of total stem volume, it may increase the bias and standard error for the taper prediction, although the amount can be small. As indicated in Table 2, the standard errors of the coefficients (*SE*) from the independent fitting method are larger than those from the simultaneous fitting method, suggesting that the estimates which result from such an algebraic procedure are not statistically efficient (Burkhart & Sprinz, 1984; Reed & Green, 1984). This is because the parameters in the total volume model are obtained by minimising the sum of squares error of total volume only, but does not ensure minimal error in taper model.

Inspection of the individual component sums of squared errors for the model showed that the simultaneous fitting method reduced the total model squared errors, which simultaneously minimised both volume and taper prediction errors. Fitting both models simultaneously also improved the fit. For example, the root mean squared error (*RMSE*) in total stem volume over bark fell from 0.033 for the "independent" method to 0.011 (66.7% in Table 2) when the simultaneous estimation was used. The corresponding decrease was 44.4% (from 0.033 to 0.012) for volume under bark (Table 2). For the taper model, when the model was fitted simultaneously the root mean squared error fell by 7.4% (from 0.040 to 0.031) and 7.8% (from 0.039 to 0.032) for upper stem diameter over bark and under bark, respectively (Table 2). Based on these results, simultaneous fitting was the more appropriate method for parameter estimation in this study. All figures, tables and discussion that follow will then be based on the parameters estimated by simultaneous fitting (second method).

B. Predicting Taper, Total and Merchantable Stem Volume

It should be noted that the compatible model of both volume and taper models was fitted using logarithmic transformation. To predict values in the original unit, a correction factor may need to be applied to correct the proportional bias in the estimate of volume or taper introduced by the back-transformation. In this study, the correction factor was calculated as the ratio of the mean of sample values to the mean of back-transformed predicted values from the regression following procedure of Snowdorn (1991), i.e. 1.0012 and 1.0029 (for total volume over and under bark, respectively), and 1.0029 and 1.0013 (for upper stem diameter over and under bark, respectively).

Taking into account the correction factors, the following models may be used to predict total volume over bark (*Vob*) and total volume under bark (*Vub*):

$$\hat{V}ob_{tot} = 0.0000636\, D^{1.736}\, Ht^{1.0734} \quad (23)$$

$$\hat{V}ub_{tot} = 0.0000542\, D^{1.778}\, Ht^{1.04} \quad (24)$$

The corresponding taper volume models that can be used to predict the diameter merchantability limit (*d*) for both over bark (*dob*) and under bark (*dub*) are:

$$\hat{d}ob = 1.384\, D^{0.8678}\, Ht^{-0.6393}\, l^{0.676} \quad (25)$$

$$\hat{d}ub = 1.256\, D^{0.889}\, Ht^{-0.6213}\, l^{0.643} \quad (26)$$

Using the compatibility relationship between the total stem volume and stem taper models, merchantable volume model were then derived.

Thus, the merchantable volume up to any desired top diameter limit (*d*) can be predicted by the following models:

$$\hat{V}ob_{mer} = 0.0000636\,D^{1.736}\,Ht^{-1.2786}\left(Ht^{2.352} - \left(0.326\,dob^{3.479}\,D^{-3.019}\,Ht^{2.224}\right)\right) \quad (27)$$

$$\hat{V}ub_{mer} = 0.0000542\,D^{1.778}\,Ht^{-1.2426}\left(Ht^{2.286} - \left(0.445\,dub^{3.555}\,D^{-3.161}\,Ht^{2.209}\right)\right) \quad (28)$$

The total stem volume may be regarded as a special case of merchantable volume when the upper stem diameter equals zero ($d = 0$). In this case, the stem volume can be predicted by either Eq.(23) or (27) for volume over bark and either Eq.(24) or (28) for volume under bark, and the results would be identical. Therefore, the merchantable volume model is also compatible with the total volume model.

An example of obtained predicted taper curves for an *A. mangium* tree with DBH = 14.5 cm and total tree height = 20 m using Eq.(25) for diameter over bark (*dob*) and Eq.(26) for diameter under bark (*dub*) are illustrated in Figure 3(a). The differences between *dob* and dub decrease with stem height, consistent with bark thickness becoming negligible at the very upper stem. The corresponding results of the stem volume curves predicted using Eq.(27) for *Vob* and Eq.(28) for *Vub* are also quite reasonable in appearance (Figure 3(b)), with the difference between the *Vob* and *Vub* being about 12.7%. For practical purpose, the prediction of volume and proportions of bark will become important as interest increases in efficient use of timber harvesting residues. Therefore, an important distinction should be made between *Vob* and *Vub*. The *Vob* model can be used to estimate the over-bark volume to be harvested and the *Vub* model can be used to estimate the under-bark volume available for utilization.

C. Model Evaluation

In addition to statistical properties presented in Table 2, the prediction accuracy of the compatible model of taper and total volume models as well as the merchantable volume model were examined based on biases and their standard errors of the predictions for both over and under bark (Table 3). The overall mean biases for both total volume and taper were found to be positive (for both over and under bark), indicating that the models are slightly under predicting. The positive mean bias was also found for merchantable volume. These results were reasonable since the merchantable volume model were derived algebraically from the compatible taper and total volume models; if the total volume and the taper (stem diameter) are underestimated then merchantable volume would be expected to be underestimated too. However, these biases and their corresponding standard errors of the predictions were

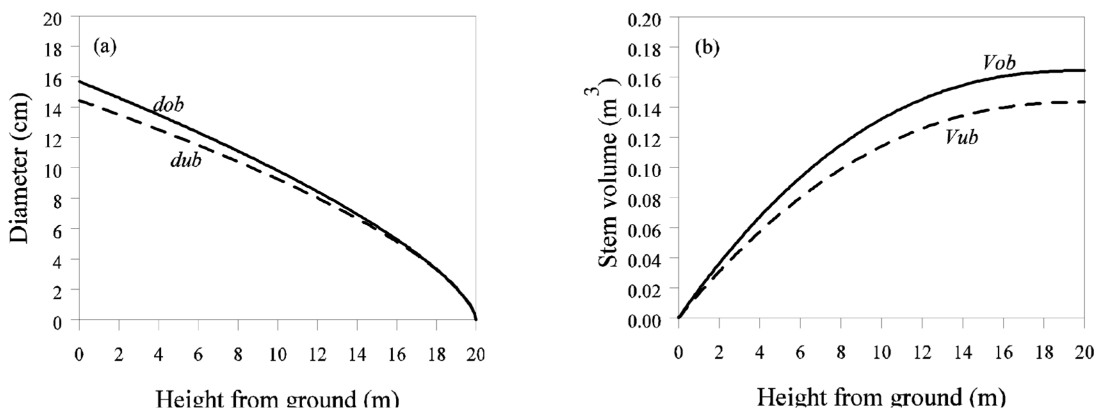

Figure 3. Examples of stem taper (a) and volume (b) curves for *A. mangium* tree with DBH = 14.5 cm and total tree height = 20 m for both over bark (*ob*) and under bark (*ub*)

Table 3. Overall mean bias and standard error of the predictions for the total volume, taper (stem diameter) and merchantable volume models

Model	Bias		Standard errors of predictions	
	over bark	under bark	over bark	under bark
Total volume (m^3)	0.0012	0.0009	0.0272	0.0135
Stem diameter (cm)	0.1019	0.0095	1.1232	0.8213
Merchantable volume (m^3)	0.0233	0.0113	0.0652	0.0381

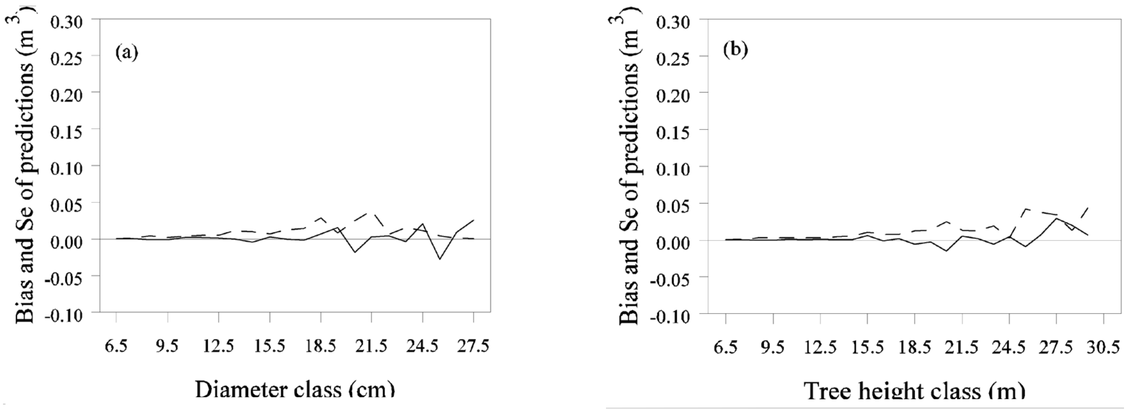

Figure 4. Average bias (*solid line*) and standard errors of the predictions (*dashed line*) for total stem volume over bark at different DBH class (a) and tree height class (b)

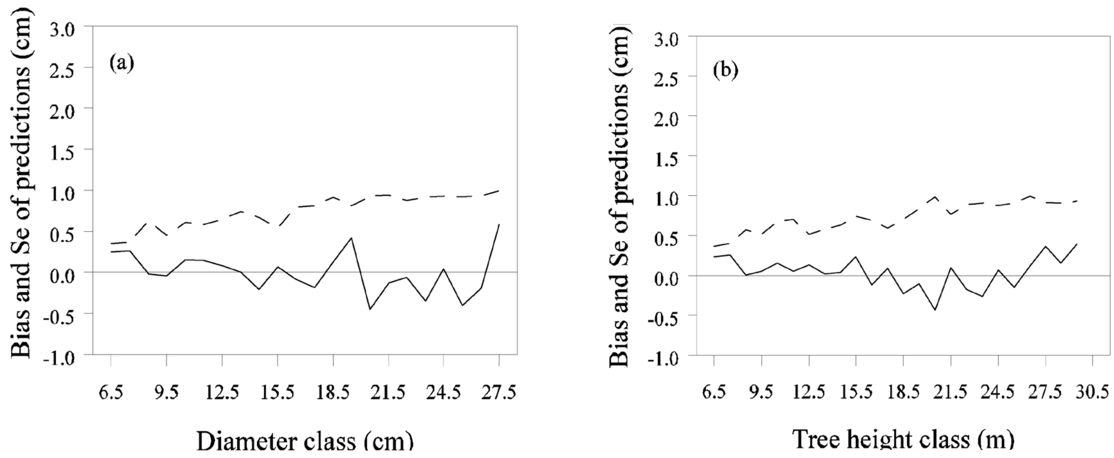

Figure 5. Average bias (*solid line*) and standard errors of the predictions (*dashed line*) for stem diameter over bark at different DBH class (a) and tree height class (b)

generally small and unlikely to be of practical importance.

Prediction accuracy was also evaluated over the entire diameter and height ranges of the data. The example of the average biases and their standard errors of the predictions for total stem volume over bark at different DBH and total tree height classes are presented in Figure 4. These graphs show that the mean biases and standard error of predictions were generally small and stable across DBH and total height classes. There were similar results (not presented

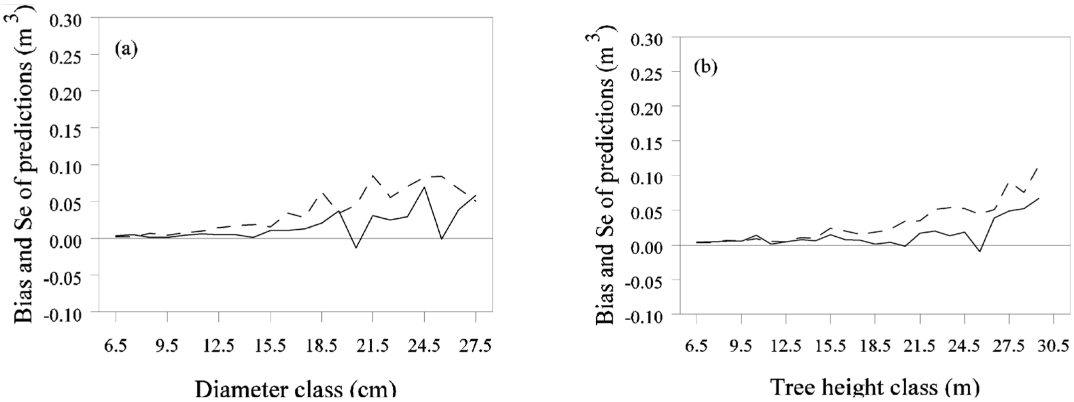

Figure 6. Average bias (*solid line*) and standard errors of the predictions (*dashed line*) for merchantable stem volume over bark at different DBH class (a) and tree height class (b)

here) for total stem volume under bark. These results suggested that both total stem volume models (Eqs.(23) and (24)) were satisfactory for predicting total stem volume over bark and stem volume under bark, respectively, with acceptable level of bias over the entire diameter and total tree height ranges of the data.

Similarly, the accuracy of the predictions for taper (stem diameter) and merchantable volume was evaluated over the ranges of DBH and total tree height classes. An example is presented for stem diameter over bark (Figure 5) and merchantable volume over bark (Figure 6). There were no strong trends in the bias and standard errors of the predictions of the stem diameters across DBH and total tree height classes although bias tended to be positive and large for the larger diameter or height classes (Figure 5). Similar pattern is evident for merchantable volume predictions. Although the biases and standard error of the predictions for merchantable volume were slightly higher for trees with larger diameter (Figure 6), this may be due to lack of data regarding bigger trees. Despite this, overall bias and standard errors of the predictions for these models were small and may be considered to be negligible for practical forest management.

These results suggested that both taper and merchantable volume models generally describe well the entire stem profile of *A. mangium* tree, and thus makes it unnecessary to develop segmented taper models for different parts of the stem as has been suggested by several authors (e.g. Max & Burkhart, 1976; Fang et al., 2000). A possible explanation is that the *A. mangium* stems used in this study did not exhibit much butt swell (less neiloidal at the lower stems), which has been shown in some other species to be the main source of the bias, either in taper or in volume models (e.g. Sharma & Zhang, 2004; Jiang et al., 2005; Rojo et al., 2005). National Research Council (1983) also reported that *A. mangium* grown in plantations usually has good stem form, with the main bole usually straight and clear, and butt swell is minimal.

D. Application for Predicting Multiple Products

Stems are usually cut to specified log lengths or diameter limits during harvesting, and more than one log may be cut from the same stem for different products. The compatible tree volume and taper model produced in this study can be used to estimate the portions of these product volumes from the total volume of a single stem. If the diameter at breast height, total tree height, and merchantable top diameter are given, estimates of the portions of these products, such as the amounts of sawtimber and pulpwood, can be calculated. The following example illustrates how Eq.(27) (for volume over bark) can be used to predict the volumes of multiple products from a single stem:

1. Suppose DBH = 28.8 cm and total tree

height = 26 m. Total stem volume over bark would be 0.7175 m^3 (estimated by Eq.(27) with top diameter, *dob* = 0).

2. If merchantable top diameter was specified as 4 cm, the merchantable volume up to 4 cm top diameter would be 0.7167 m^3 (estimated by Eq.(27) with top diameter, *dob* = 4).
3. Merchantable volume specified for sawtimber (e.g. dob = 20 cm) would be 0.5141 m^3 (estimated by Eq.(27) with top diameter, *dob* = 20).
4. The difference between total volume and merchantable volume of sawtimber could be the volumes of pulpwood or fuel wood. If pulpwood is specified to be 4 cm in top diameter limit, the volume of pulpwood would lie between the 20 and 4 cm top diameter limits (i.e. 0.7167 – 0.5141 = 0.2026 m^3).
5. The remaining volume (between 4 and 0 cm in top diameter limit) could be the fuel wood or stay in the stand as residue after harvest (i.e. 0.0008 m^3).

Similar procedures can be repeated to predict volume under bark using Eq.(28).

The above examples demonstrate the advantage of estimating volume through taper models over existing volume models or volume tables. This is due to the ability of taper models to accurately predict diameter over bark or diameter under bark at any given height of individual trees, hence allowing the acquisition of merchantable volume estimates to any desired specification. This benefit has also been reported by Li and Weiskittel (2010).

IV. CONCLUSION

The best compatible model for estimating stem volume and taper developed for *A. mangium* plantations was derived based on the logarithmic function fitted using a simultaneous method of the seemingly unrelated regression. The developed compatible model is efficient and flexible enough to estimate total stem volume, merchantable volume to any merchantability limit, diameter at any height, and (possibly) height of any diameter based on only easily measurable parameters such as diameter at breast height and total tree height. Predicted volumes to various merchantability limits can be obtained using a single model for volume over bark (*Vob*) and volume under bark (*Vub*), respectively, as follows:

$$\hat{V}ob_{mer} = 0.0000636\, D^{1.736} Ht^{-1.2786} \left(Ht^{2.352} - \left(0.326\, dob^{3.479} D^{-3.019} Ht^{2.224} \right) \right)$$

$$\hat{V}ub = 0.0000542\, D^{1.778} \left(Ht^{2.286} - \left(0.445\, dub^{3.555} D^{-3.161} Ht^{2.209} \right) \right)$$

The model has been shown to perform well for the two data sets (over and under bark measurements) of the total volume as well as merchantable volume. The taper model appears to be sufficient to describe the stem profile of *A. mangium* trees, thus eliminating the need to develop segmented taper models for different parts of the stem.

In stand yield prediction where yield estimates by product category are required, the model will prove to be very useful for estimating total and merchantable volumes of individual trees and thus the total and merchantable volumes expected from the stands. The compatible model produced in this study also provide a major improvement over the previous models for *A. mangium* plantations in Indonesia which were mainly developed based on different standard volume models and is only applicable for estimating stem volume to a fixed top diameter limit.

ACKNOWLEDGEMENTS

The author thanks to the staff of PT. Musi Hutan Persada for field assistance and data collection. Valuable comments from Dr. Peter Ades and Dr. Yue Wang on the earlier draft of this manuscript, and two anonymous reviewers for their constructive comments and suggestions are gratefully acknowledged.

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